Ontario eSecondary School
Course Outline
2023-2024

Ministry of Education Course Title: Advanced Functions, University Preparation	
Ministry Course Code: MHF4U	
Course Type: University Preparation	
Grade: 12	
Credit Value: 1.0	
Prerequisite(s): MCR3U Functions, Grade 11, University Preparation	
Department: Mathematics	
Course developed by: **Marieta Angjeli**	**Date: September 1, 2017** **Revised: March 1, 2019**
Length: **One Semester**	**Hours:** **110**
This course has been developed based on the following Ministry documents: 1. *The Ontario Curriculum, Grades 11 and 12 Mathematics, Revised 2007* 2. *Growing Success: Assessment, Evaluation, and Reporting in Ontario Schools (2010)* 3. *Learning for All (2013)*	

COURSE DESCRIPTION/RATIONALE

This course extends students' experience with functions. Students will investigate the properties of polynomial, rational, logarithmic, and trigonometric functions; develop techniques for combining functions; broaden their understanding of rates of change; and develop facility in applying these concepts and skills. Students will also refine their use of the mathematical processes necessary for success in senior mathematics. This course is intended both for students taking the Calculus and Vectors course as a prerequisite for a university program and for those wishing to consolidate their understanding of mathematics before proceeding to any one of a variety of university programs.

OVERALL CURRICULUM EXPECTATIONS

Unit 1:
By the end of this course, students will:

- identify and describe some key features of polynomial functions, and make connections between the numeric, graphical, and algebraic representations of polynomial functions;
- solve problems involving polynomial graphically and algebraically;
- demonstrate an understanding of solving polynomial inequalities.

Unit 2:
By the end of this course, students will:

- identify and describe some key features of the graphs of rational functions, and represent rational functions graphically;
- solve problems involving simple rational equations graphically and algebraically;
- demonstrate an understanding of solving simple rational inequalities.

Unit 3:
By the end of this course, students will:

- demonstrate an understanding of the relationship between exponential expressions and logarithmic expressions, evaluate logarithms, and apply the laws of logarithms to simplify expressions;
- identify and describe some key features of the graphs of logarithmic functions, make connections among the numeric, graphical, and algebraic representations of logarithmic functions, and solve related problems graphically;
- solve exponential and simple logarithmic equations in one variable algebraically, including those in problems arising from real-world applications.

Unit 4:
By the end of this course, students will:

- demonstrate an understanding of the meaning and application of radian measure;
- make connections between trigonometric ratios and the graphical and algebraic representations of the corresponding trigonometric functions and between trigonometric functions and their reciprocals, and use these connections to solve problems;
- solve problems involving trigonometric equations and prove trigonometric identities.

Unit 5:
By the end of this course, students will:

- demonstrate an understanding of average and instantaneous rate of change, and determine, numerically and graphically, and interpret the average rate of change of a function over a given interval and the instantaneous rate of change of a function at a given point;
- determine functions that result from the addition, subtraction, multiplication, and division of two functions and from the composition of two functions, describe some properties of the resulting functions, and solve related problems;

- compare the characteristics of functions, and solve problems by modeling and reasoning with functions, including problems with solutions that are not accessible by standard algebraic techniques.

COURSE CONTENT

Unit	*Length*
1: Polynomial Functions	25 hours
2: Rational Functions	15 hours
3: Exponential and Logarithmic Functions	17 hours
4: Trigonometric Functions	32 hours
5: Algebra of Functions and Rates of Change	15 hours
Culminating Project	4 hours
Final Exam	2 hours
Total	110 hours

UNIT DESCRIPTIONS

Unit 1 - Polynomial Functions
Students will investigate polynomial functions. They will extend their knowledge about linear and quadratic functions to include cubic, quartic and quintic functions. Students will explore their graphs and characteristics, also distinguish polynomial functions from sinusoidal and exponential functions, and compare and contrast the graphs of various polynomial functions with the graphs of other types of functions. They will develop skills in how to factorize polynomial functions to the 5^{th} degree and graph these functions with transformation applied. Students will determine, through investigation with and without technology, key features (i.e. domain and range, intercepts, positive/negative intervals, increasing/decreasing intervals) of the graphs of polynomial functions. Students will solve problems involving applications of polynomial functions and equations and explain the difference between the solution to an equation in one variable and the solution to an inequality in one variable, also demonstrate that given solutions satisfy an inequality and determine solutions to polynomial inequalities in one variable by graphing the corresponding functions, using graphing technology, and identifying intervals for which x satisfies the inequalities.

Unit 2 - Rational functions
Students will investigate rational functions. Students will determine, through investigation with and without technology, key features (i.e., vertical and horizontal asymptotes, domain and range, intercepts, positive/negative intervals, increasing/decreasing intervals) of the graphs of rational functions that are the reciprocals of linear and quadratic functions, and make connections between the algebraic and graphical representations of these rational functions. Students will solve problems involving applications of simple rational functions and equations and explain the difference between the solution to an equation in one variable and the solution to an inequality in one variable, also demonstrate that given solutions satisfy an inequality and determine solutions to simple rational inequalities in one variable by graphing the corresponding functions, using graphing technology, and identifying intervals for which x satisfies the inequalities

Unit 3- Exponential and Logarithmic Functions
Students make connections between related logarithmic and exponential equations and between the laws of exponents and the laws of logarithms, verify the laws of logarithms with or without technology, and use the laws of logarithms to simplify and evaluate numerical expressions. Students will also solve problems

involving exponential and logarithmic equations algebraically, including problems arising from real-world applications.

Unit 4 - Trigonometric Functions
Students recognize the radian as an alternative unit to the degree for angle measurement, define the radian measure of an angle as the length of the arc that subtends this angle at the centre of a unit circle, and develop and apply the relationship between radian and degree measure. Students will also sketch the graphs of f(x) = sinx and f(x) = cosx for angle measures expressed in radians, and determine and describe some key properties (e.g., period of 2 π , amplitude of 1) in terms of radians. Students represent a sinusoidal function with an equation, given its graph or its properties, with angles expressed in radians. Students recognize that trigonometric identities are equations that are true for every value in the domain, prove trigonometric identities through the application of reasoning skills, using a variety of relationships, and verify identities using technology.

Unit 5 - Algebra of Functions and Rates of Change
Students extend their knowledge about characteristics of different functions to key features as domain, range, maximum/minimum points, number of zeros of the graphs of functions created by adding, subtracting, multiplying, or dividing functions, and describe factors that affect these properties. Students will also investigate the composition of two functions [i.e., f(g(x))] numerically (i.e., by using a table of values) and graphically, with technology, for functions represented in a variety of ways (e.g., function machines, graphs, equations), and interpret the composition of two functions in real-world applications. Students make connections, through investigation, between the slope of a secant on the graph of a function (e.g., quadratic, exponential, sinusoidal) and the average rate of change of the function over an interval, and between the slope of the tangent to a point on the graph of a function and the instantaneous rate of change of the function at that point.

TEACHING AND LEARNING STRATEGIES

In this course, students will experience the following activities.

Presentations with embedded videos are utilized to outline concepts, explain theory with the use of examples and practice questions, and incorporate multi-media opportunities for students to learn more (e.g. online simulations, quizzes, etc.).

End of unit conversations and Poodlls are opportunities for students to express their ideas, problem solving, and thought processes with a teacher who provides timely feedback.

Reflection is an opportunity for students to look back at concepts and theories with new eyes, to relate theory to practice, and to align learning with their own values and beliefs.

Discussions with the instructor are facilitated through video conferencing, discussing the concepts and skills being studied. This enables two-way communication between the student and the instructor, to share ideas and ask questions in dialogue. This also helps to build a relationship between the student and instructor.

Instructor demonstrations (research skills, etc.) are opportunities for the instructor to lead a student through a concept or skill through video conferencing, videos, or emailing with the student.

Discussion forums are an opportunity for students to summarize and share their ideas and perspectives with their peers, which deepens understanding through expression. It also provides an opportunity for peer-to-peer feedback.

Practical extension and application of knowledge are integrated throughout the course. The goal is to help students make connections between what they learn in the classroom and how they understand and relate to the world around them and their own lives. Learning becomes a dynamic opportunity for students to be more aware that their learning is all around them and enable them to create more meaning in their lives.

Individual activities/assignments assessments are completed individually at a student's own pace and are intended to expand and consolidate the learning in each lesson. Individual activities allow the teacher to accommodate interests and needs and to assess the progress of individual students. For this reason, students are encouraged to discuss IEPs (Individual Education Plans) with their teacher and to ask to modify assessments if they have a unique interest that they feel could be pursued in the assessment. The teacher plays an important role in supporting these activities by providing ongoing feedback to students, both orally and in writing.

Research is an opportunity to apply inquiry skills to a practical problem or question. Students perform research to gather information, evaluate quality sources, analyze findings, evaluate their analysis, and synthesize their findings into conclusions. Throughout, students apply both creative thinking and critical thinking. New questions are also developed to further learning.

Writing as a learning tool helps students to think critically about course material while grasping, organizing, and integrating prior knowledge with new concepts. Good communication skills are important both in and out of the classroom.

Virtual simulations are interactive websites that provide students with an opportunity to ask questions, relate variables, and examine relationships.

Diagrams are visual representations of mathematical ideas and concepts. They provide another perspective to organize ideas. Visuals are thought to promote cognitive plasticity - meaning, they can help us change our minds or help us to remember an idea.

Graphs and charts are visual representations of math concepts and analysis. This helps us to see the relationships within and between sets of data.

Tables involve organizing information in terms of categories (rows and columns). This helps us to understand the relationships between ideas and data, as well as highlight trends.

Practice problems provide students with a scenario/problem to solve by applying concepts and skills learned in a context. This helps students to understand the relevance of their learning.

ASSESSMENT, EVALUATION, AND REPORTING

Assessment: The process of gathering information that accurately reflects how well a student is achieving the identified curriculum expectations. Teachers provide students with descriptive feedback that guides their efforts towards improved performance.

Evaluation: Assessment of Learning focuses on Evaluation which is the process of making a judgement about the quality of student work on the basis of established criteria over a limited, reasonable period of time.

Reporting: Involves communicating student achievement of the curriculum expectations and Learning Skills and Work Habits in the form of marks and comments as determined by the teacher's use of professional judgement.

STRATEGIES FOR ASSESSMENT

Assessment practices can nurture students' sense of progress and competency and information instruction. Many diagnostic tools, e.g. checklists, are used at regular intervals throughout the units to encourage students' understanding of their current status as learners and to provide frequent and timely reviews of their progress. Assessment of student acquisition of learning skills also occurs regularly through unobtrusive teacher observation and conferencing.

Teachers are encouraged to share goals with students early in the course and to connect unit learning experiences frequently and explicitly with big ideas, overall expectations, and performance tasks. The

teacher is encouraged to involve students in the discussion, modification, or creation of rubrics, and teach students to use rubrics as a learning tool..

ASSESSMENT ACTIVITIES

- ❑ Homework assignments
- ❑ Individual conference meetings
- ❑ Discussion Forums
- ❑ Diagnostic tests and writing tasks
- ❑ Completed Templates & Graphic Organizers
- ❑ Reflections
- ❑ Oral presentations & Active Listening
- ❑ Tests & Exam
- ❑ Evaluations

EVALUATION

The final grade will be determined as follows:

- ❑ 70% of the grade will be based on evaluation conducted throughout the course. This portion of the grade should reflect the student's most consistent level of achievement throughout the course, although special consideration will be given to more recent evidence of achievement.
- ❑ 30% of the grade will be based on a final evaluation administered at or towards the end of the course. This evaluation will be based on evidence from one or a combination of the following: an examination, a performance, and/or another method of evaluation suitable to the course content. The final evaluation allows the student an opportunity to demonstrate comprehensive achievement of the overall expectations for the course.

  (*Growing Success: Assessment, Evaluation and Reporting in Ontario Schools*. Ontario Ministry of Education Publication, 2010 p.41)

	Weight
Course Work	**70**
Knowledge/Understanding (K)	20
Thinking/Inquiry (T)	15
Communication (C)	15
Application (A)	20
Final	**30**
Exam (5K, 5T, 5C, 5A)	20
Culminating Project (2.5K, 2.5T, 2.5C, 2.5A)	10

TERM WORK EVALUATIONS (70%)

Evaluation Item	Description	Category
Reflective research projects	Research-based projects for each unit based on applications of learned functions to real life problems	K,T,C,A

Problem sets	Problem sets supplement lessons and are used to assess whether or not students are meeting criteria for success	K,T,C
Simulations/Live interview evaluations	Live interviews are used to evaluate students through observation and conversation. Some of these are a part of other research projects.	K,T,C,A
Unit Test(s)	Unit tests are based on curriculum expectations and cover the entirety of each unit	K,T,C,A

FINAL EVALUATIONS (30%)

Evaluation Item	Description	Category
Final Exam 20%	A final, written examination, covering all curriculum expectations for the course.	K,T,C,A
Culminating Project 10%	A comprehensive project, covering all overall curriculum expectations for the course.	K,T,C,A

AFL/AAL/AOL Tracking Sheet

Unit 1:

AAL	**AFL**	**AOL**
Discussion Forum	Gizmos Investigation	Assignment: Quiz on remainder and factor theorem
Assignment: H5P Factoring Sum and Cubes	Online quiz	Unit Project
Assignment: You be the teacher!	Lesson Problem Set [Self-Check Answers]	Unit Test

Unit 2:

AAL	**AFL**	**AOL**
Discussion Forum	Gizmos Investigation	Assignment: Quiz on Rational Functions
Assignment: Problem set	Online quiz	Unit Project
Assignment: Create Rational Functions POODLL	Lesson Problem Set [Self-Check Answers]	Unit Test

Unit 3:

AAL	**AFL**	**AOL**
Discussion Forum	Gizmos Investigation	Assignment: Quiz on Logarithmic Functions
Assignment: Problem set	Online quiz	Unit Project
Assignment: Laws of logarithms POODLL	Lesson Problem Set [Self-Check Answers]	Unit Test

Unit 4:

AAL	**AFL**	**AOL**
Discussion Forum	Gizmos Investigation	Assignment: Quiz on graphs of trig functions
Assignment: Problem set	Online quiz	Unit Project
Assignment: Model with Trig Functions POODLL	Lesson Problem Set [Self-Check Answers]	Unit Test

Unit 5:

AAL	**AFL**	**AOL**
Discussion Forum	Investigation	Assignment: Quiz on Composite Functions
Assignment: Problem set	Online quiz	Unit Project
Assignment: Match graphs and equations POODLL	Lesson Problem Set [Self-Check Answers]	Unit Test

Finals

AOL
Culminating Activity
Final Exam

CONSIDERATION FOR PROGRAM PLANNING

PLANNING PROGRAMS FOR STUDENTS WITH SPECIAL EDUCATION NEEDS

Classroom teachers are the key educators of students who have special education needs. They have a responsibility to help all students learn, and they work collaboratively with special education teachers, where appropriate, to achieve this goal. Special Education Transformation: The Report of the Co-Chairs with the Recommendations of the Working Table on Special Education, 2006 endorses a set of beliefs that should guide program planning for students with special education needs in all disciplines. Those beliefs are as follows: All students can succeed. Universal design and differentiated instruction are effective and interconnected means of meeting the learning or productivity needs of any group of students. Successful instructional practices are founded on evidence-based research, tempered by experience.

PROGRAM CONSIDERATIONS FOR ENGLISH LANGUAGE LEARNERS

Ontario schools have some of the most multilingual student populations in the world. The first language of approximately 20 percent of the students in Ontario's English language schools is a language other than English. Ontario's linguistic heritage includes several Aboriginal languages; many African, Asian, and European languages; and some varieties of English, such as Jamaican Creole. Many English language learners were born in Canada and raised in families and communities in which languages other than English were spoken, or in which the variety of English spoken differed significantly from the English of Ontario classrooms. Other English language learners arrive in Ontario as newcomers from other countries; they may have experience of highly sophisticated educational systems, or they may have come from regions where access to formal schooling was limited. When they start school in Ontario, many of these students are entering a new linguistic and cultural environment.

THE ROLE OF TECHNOLOGY IN THE PROGRAM

Information and communications technologies (ICT) provide a range of tools that can significantly extend and enrich teachers' instructional strategies and support students' language learning. ICT tools include multimedia resources, databases, Internet websites, digital cameras, and word-processing programs. Tools such as these can help students to collect, organize, and sort the data they gather and to write, edit, and present reports on their findings. Information and communications technologies can also be used to connect students to other schools, at home and abroad, and to bring the global community into the local classroom. Whenever appropriate, therefore, students should be encouraged to use ICT to support and communicate their learning.

ACCOMMODATIONS

Accommodations will be based on meeting with parents, teachers, administration and external educational assessment reports. The following three types of accommodations may be provided:

- ***Instructional accommodations:*** such as changes in teaching strategies, including styles of presentation, methods of organization, or use of technology and multimedia.
- ***Assessment accommodations:*** such as allowing additional time to complete tests or assignments or permitting oral responses to test questions.

Other examples of modifications and aids, which may be used in this course, are:

- Provide step-by-step instructions.
- Help students create organizers for planning writing tasks.
- Allow students to report verbally to a scribe (teacher/ student) who can help in note taking.
- Permit students a range of options for reading and writing tasks.
- Where an activity requires reading, provide it in advance.
- Provide opportunities for enrichment.