Ontario eSecondary School
Course Outline
2022-2023

Ministry of Education Course Title: Advanced Functions, University Preparation	
Ministry Course Code: MHF4U	
Course Type: University Preparation	
Grade: 12	
Credit Value: 1.0	
Prerequisite(s): MCR3U Functions, Grade 11, University Preparation	
Department: Mathematics	
Course developed by: **Marieta Angjeli**	**Date: September 1, 2017** **Revised: March 1, 2019**
Length: **One Semester**	**Hours:** **110**
This course has been developed based on the following Ministry documents: 1. *The Ontario Curriculum, Grades 11 and 12 Mathematics, Revised 2007* 2. *Growing Success: Assessment, Evaluation, and Reporting in Ontario Schools (2010)*	

COURSE DESCRIPTION/RATIONALE

This course extends students' experience with functions. Students will investigate the properties of polynomial, rational, logarithmic, and trigonometric functions; develop techniques for combining functions; broaden their understanding of rates of change; and develop facility in applying these concepts and skills. Students will also refine their use of the mathematical processes necessary for success in senior mathematics. This course is intended both for students taking the Calculus and Vectors course as a prerequisite for a university program and for those wishing to consolidate their understanding of mathematics before proceeding to any one of a variety of university programs.

OVERALL CURRICULUM EXPECTATIONS

Unit 1:
By the end of this course, students will:

- identify and describe some key features of polynomial functions, and make connections between the numeric, graphical, and algebraic representations of polynomial functions;
- solve problems involving polynomial graphically and algebraically;
- demonstrate an understanding of solving polynomial inequalities.

Unit 2:
By the end of this course, students will:

- identify and describe some key features of the graphs of rational functions, and represent rational functions graphically;
- solve problems involving simple rational equations graphically and algebraically;
- demonstrate an understanding of solving simple rational inequalities.

Unit 3:
By the end of this course, students will:

- demonstrate an understanding of the relationship between exponential expressions and logarithmic expressions, evaluate logarithms, and apply the laws of logarithms to simplify expressions;
- identify and describe some key features of the graphs of logarithmic functions, make connections among the numeric, graphical, and algebraic representations of logarithmic functions, and solve related problems graphically;
- solve exponential and simple logarithmic equations in one variable algebraically, including those in problems arising from real-world applications.

Unit 4:
By the end of this course, students will:

- demonstrate an understanding of the meaning and application of radian measure;
- make connections between trigonometric ratios and the graphical and algebraic representations of the corresponding trigonometric functions and between trigonometric functions and their reciprocals, and use these connections to solve problems;
- solve problems involving trigonometric equations and prove trigonometric identities.

Unit 5:
By the end of this course, students will:

- demonstrate an understanding of average and instantaneous rate of change, and determine, numerically and graphically, and interpret the average rate of change of a function over a given interval and the instantaneous rate of change of a function at a given point;
- determine functions that result from the addition, subtraction, multiplication, and division of two functions and from the composition of two functions, describe some properties of the resulting functions, and solve related problems;
- compare the characteristics of functions, and solve problems by modeling and reasoning with functions, including problems with solutions that are not accessible by standard algebraic techniques.

COURSE CONTENT

Unit	*Length*
1: Polynomial Functions	25 hours
2: Rational Functions	15 hours
3: Exponential and Logarithmic Functions	17 hours
4: Trigonometric Functions	32 hours
5: Algebra of Functions and Rates of Change	15 hours
Culminating Project	4 hours
Final Exam	2 hours

UNIT DESCRIPTIONS

Unit 1 - Polynomial Functions

Students will investigate polynomial functions. They will extend their knowledge about linear and quadratic functions to include cubic, quartic and quintic functions. Students will explore their graphs and characteristics, also distinguish polynomial functions from sinusoidal and exponential functions, and compare and contrast the graphs of various polynomial functions with the graphs of other types of functions. They will develop skills in how to factorize polynomial functions to the 5^{th} degree and graph these functions with transformation applied. Students will determine, through investigation with and without technology, key features (i.e. domain and range, intercepts, positive/negative intervals, increasing/decreasing intervals) of the graphs of polynomial functions. Students will solve problems involving applications of polynomial functions and equations and explain the difference between the solution to an equation in one variable and the solution to an inequality in one variable, also demonstrate that given solutions satisfy an inequality and determine solutions to polynomial inequalities in one variable by graphing the corresponding functions, using graphing technology, and identifying intervals for which x satisfies the inequalities.

Unit 2 - Rational functions

Students will investigate rational functions. Students will determine, through investigation with and without technology, key features (i.e., vertical and horizontal asymptotes, domain and range, intercepts, positive/negative intervals, increasing/decreasing intervals) of the graphs of rational functions that are the reciprocals of linear and quadratic functions, and make connections between the algebraic and graphical representations of these rational functions. Students will solve problems involving applications of simple rational functions and equations and explain the

difference between the solution to an equation in one variable and the solution to an inequality in one variable, also demonstrate that given solutions satisfy an inequality and determine solutions to simple rational inequalities in one variable by graphing the corresponding functions, using graphing technology, and identifying intervals for which x satisfies the inequalities

Unit 3- Exponential and Logarithmic Functions

Students make connections between related logarithmic and exponential equations and between the laws of exponents and the laws of logarithms, verify the laws of logarithms with or without technology, and use the laws of logarithms to simplify and evaluate numerical expressions. Students will also solve problems involving exponential and logarithmic equations algebraically, including problems arising from real-world applications.

Unit 4 - Trigonometric Functions

Students recognize the radian as an alternative unit to the degree for angle measurement, define the radian measure of an angle as the length of the arc that subtends this angle at the centre of a unit circle, and develop and apply the relationship between radian and degree measure. Students will also sketch the graphs of f(x) = sinx and f(x) = cosx for angle measures expressed in radians, and determine and describe some key properties (e.g., period of 2π , amplitude of 1) in terms of radians. Students represent a sinusoidal function with an equation, given its graph or its properties, with angles expressed in radians. Students recognize that trigonometric identities are equations that are true for every value in the domain, prove trigonometric identities through the application of reasoning skills, using a variety of relationships, and verify identities using technology.

Unit 5 - Algebra of Functions and Rates of Change

Students extend their knowledge about characteristics of different functions to key features as domain, range, maximum/minimum points, number of zeros of the graphs of functions created by adding, subtracting, multiplying, or dividing functions, and describe factors that affect these properties. Students will also investigate the composition of two functions [i.e., f(g(x))] numerically (i.e., by using a table of values) and graphically, with technology, for functions represented in a variety of ways (e.g., function machines, graphs, equations), and interpret the composition of two functions in real-world applications. Students make connections, through investigation, between the slope of a secant on the graph of a function (e.g., quadratic, exponential, sinusoidal) and the average rate of change of the function over an interval, and between the slope of the tangent to a point on the graph of a function and the instantaneous rate of change of the function at that point.

TEACHING AND LEARNING STRATEGIES

The students will experience a variety of activities:

Whole-Class Activities (facilitated through discussion forums)

Whole class activities are designed to introduce concepts and skills that are directly applicable to the workplace and to build on the content being studied during small group and individual activities. These activities include the following:

Teacher demonstrations through video conferencing, email, or videos provided of a teacher or student demonstrating the concepts and skills being studied. This helps the student and teacher create an atmosphere of trust and respect to aid in the online learning environment.

Video presentations and technological aids with videos embedded to enrich the course content and clarify concepts and skills being studied. Also the use of online pre-approved quizzes and games to help a student become more familiar with the concepts and skills being studied.

Diagnostic and review activities (audio and video recordings) can be student-lead or teacher lead to work as a review for students through audio and video made to share among each other to help reinforce the concepts and skills being studied.

Small Group Activities

The teacher sets up small group activities to provide opportunities for active and oral learning as well as to bolster practical communication and teamwork skills. The teacher plays a critical role during group activities by monitoring group progress as well as answering questions that arise and using questions to assist students in their understanding. In this way, the teacher also facilitates student understanding of effective learning, communication, and team building during group activities.

The small group activities include the following:

Paired or small group research activities students are able to share their work online with not only their teachers, but their classmates too. Students are able to share resources through online chat and video conferencing. The ability to learn from each other, work on teamwork skills, and practice communication are valued and encouraged throughout the course.

Comparison and evaluation of written work is very important in this course. This course focuses on giving many examples of correct work, and helping students build the skills needed to peer-correct and self-correct. Students are given a variety of texts to read through embedded links, to make comparisons with different texts, real life situations, and their own writing.

Practical extension and application of knowledge is used as an effective learning strategy in this course because it allows the students to read and listen to the texts and stories and reflect back with connections to themselves, other texts and the world. Students are encouraged to share their understandings through work submitted each day, phone conversations about course work, or videoconferencing.

Oral presentations in an online environment we have the equipment to have student either live video conference oral presentations, or make videos and submit them for their oral presentations. These oral presentations can be viewed by fellow students (when appropriate) and the teacher. Students can learn from one another, and from their teacher. Such activities include dramatic readings and performances.

Charts and graphs are used to present effective learning opportunities of concepts and skills to students who would benefit from visual objects to learn. Every student learns differently, and it is used to help students discover another way to present their information such as graphic organizers, lists, and pictures.

Individual Activities

The teacher should provide a variety of individual assignments to expand and consolidate the learning that takes place in the whole-class and small group activities. Individual activities allow the teacher to accommodate interests and needs and to access the progress of individual students. The teacher plays an important role in supporting these activities through the provision of ongoing feedback to the students, both orally and in writing. Teachers are encouraged to include individual activities such as the following in the course:

Research is completed in an online environment by teaching the students first about plagiarism rules and giving examples of good sources to use. The students are not only limited to the online search for

information, but have resources available by links on the Moodle page of information that has been scanned and uploaded.

Individual assignments are worked on at a student's own pace. The teacher can support the student in these activities with ongoing feedback.

Oral presentations are facilitated through the use of video conferencing and video recording.

Practical extension and application of knowledge helps students develop their own voice, and gives them the ability to make personal connections, and connections to the world throughout their course. Students are given a variety or reading and viewing texts to give them many chances to apply their new concepts, skills, and knowledge.

Ongoing project work is something that is valued in the earning of an English credit. The ongoing project can be submitted to the teacher for ongoing feedback in both written and oral work.

ASSESSMENT, EVALUATION, AND REPORTING

Assessment: The process of gathering information that accurately reflects how well a student is achieving the identified curriculum expectations. Teachers provide students with descriptive feedback that guides their efforts towards improved performance.

Evaluation: Assessment of Learning focuses on Evaluation which is the process of making a judgement about the quality of student work on the basis of established criteria over a limited, reasonable period of time.

Reporting: Involves communicating student achievement of the curriculum expectations and Learning Skills and Work Habits in the form of marks and comments as determined by the teacher's use of professional judgement.

STRATEGIES FOR ASSESSMENT

Assessment practices can nurture students' sense of progress and competency and information instruction. Many diagnostic tools, e.g. checklists, are used at regular intervals throughout the units to encourage students' understanding of their current status as learners and to provide frequent and timely reviews of their progress. Assessment of student acquisition of learning skills also occurs regularly through unobtrusive teacher observation and conferencing.

Teachers are encouraged to share goals with students early in the course and to connect unit learning experiences frequently and explicitly with big ideas, overall expectations, and performance tasks. The teacher is encouraged to involve students in the discussion, modification, or creation of rubrics, and teach students to use rubrics as a learning tool..

ASSESSMENT ACTIVITIES

- ❑ Homework assignments
- ❑ Individual conference meetings
- ❑ Discussion Forums
- ❑ Diagnostic tests and writing tasks
- ❑ Completed Templates & Graphic Organizers
- ❑ Reflections
- ❑ Oral presentations & Active Listening

- ❑ Tests & Exam
- ❑ Evaluations

EVALUATION

The final grade will be determined as follows:

- ❑ 70% of the grade will be based on evaluation conducted throughout the course. This portion of the grade should reflect the student's most consistent level of achievement throughout the course, although special consideration will be given to more recent evidence of achievement.
- ❑ 30% of the grade will be based on a final evaluation administered at or towards the end of the course. This evaluation will be based on evidence from one or a combination of the following: an examination, a performance, and/or another method of evaluation suitable to the course content. The final evaluation allows the student an opportunity to demonstrate comprehensive achievement of the overall expectations for the course.

  (*Growing Success: Assessment, Evaluation and Reporting in Ontario Schools*. Ontario Ministry of Education Publication, 2010 p.41)

	Weight
Course Work	**70**
Knowledge/Understanding	20
Thinking/Inquiry	15
Communication	15
Application	20
Final	**30**
Exam	20
Culminating Project	10

TERM WORK EVALUATIONS (70%):

Evaluation Item	Description	Category
Reflective research projects	Research-based projects for each unit based on applications of learned functions to real life problems	K,T,C,A
Problem sets	Problem sets supplement lessons and are used to assess whether or not students are meeting criteria for success	K,T,C
Simulations/Live interview evaluations	Live interviews are used to evaluate students through observation and conversation. Some of these are a part of other research projects.	K,T,C,A
Unit Test(s)	Unit tests are based on curriculum expectations and cover the entirety of each unit	K,T,C,A

FINAL EVALUATIONS (30%):

Evaluation Item	Description	Category
Final Exam 20%	A final, written examination, covering all curriculum expectations for the course.	K,T,C,A
Culminating Project 10%	A comprehensive project, covering all overall curriculum expectations for the course.	K,T,C,A

AFL/AAL/AOL Tracking sheet:

Unit 1: Polynomial functions

AAL	**AFL**	**AOL**
Lesson 1.1 notes/problem sets	Assignment 1.1 Intro to Polynomial Functions	1.4 TIMED - Quiz Remainder and factor theorem
Lesson 1.2 notes/problem sets	Assignment 1.2 Characteristics of Polynomial Functions	1.6 Assignment: You be the teacher!
Lesson 1.3 notes/problem sets	1.5 Factoring Sums and Differences of Cubes Assignment	Unit 1 Culminating Assignment
Lesson 1.4 notes/problem sets	1.7 Assignment - Polynomial Inequalities	Unit 1: End of Unit Question Response
Lesson 1.5 notes/problem sets	Unit 1: End of Unit Feedback	Unit Test
Lesson 1.6 notes/problem sets		
Lesson 1.7 notes/problem sets		

Unit 2: Rational Functions

AAL	AFL	AOL
Lesson 2.1 notes/problem sets	Assignment 2.1 Graphs of rational functions	2.3 Assignment: Create a rational Function
Lesson 2.2 notes/problem sets	Assignment 2.2 Asymptotes	Unit 2 Culminating Assignment
Lesson 2.3 notes/problem sets	Assignment 2.4 Rational Equations	Unit Test
Lesson 2.4 notes/problem sets	Assignment 2.5 Rational Inequalities	
Lesson 2.5 notes/problem sets	Unit 2: End of Unit Feedback	

Unit 3: Exponential and Logarithmic Functions

AAL	AFL	AOL
Lesson 3.1 notes/problem sets	Assignment 3.1 Exponential Functions	Timed Quiz on Log, Functions, Transformations and Laws of Logarithms
Lesson 3.2 notes/problem sets	Assignment 3.2 Logarithms	Unit 3 culminating assessment
Lesson 3.3 notes/problem sets	Assignment 3.4 Solving Exponential Equations	Unit 3: End of Unit Question Response
Lesson 3.4 notes/problem sets	Assignment 3.4 Solving Logarithmic Equations	Unit Test
	Unit 3: End of Unit Feedback	

Unit 4: Trigonometric Functions

AAL	AFL	AOL
Lesson 4.1 notes/problem sets	4.1 assignment - quiz radians measures	4.3 Assignment Graphs of Reciprocal Functions (Timed Quiz)

Lesson 4.2 notes/problem sets	4.2 assignment - trigonometric exact values	4.7 Assignment - Compound and Double Angle Formulas (Timed Quiz)
Lesson 4.3 notes/problem sets	4.4 Assignment - Transformations of Trigonometric Functions	Unit 4 Culminating assignment
Lesson 4.4 notes/problem sets	Additional Practice - Transformations of trigonometric functions	Unit Test
Lesson 4.5 notes/problem sets	4.5 Assignment Modelling with Trigonometric Functions	
Lesson 4.6 notes/problem sets	4.6 Assignment Equivalent Trigonometric Expressions	
Lesson 4.7 notes/problem sets	4.7 Additional Practice - Compound Angle Formulas with answers	
Lesson 4.8 notes/problem sets	4.8 Assignment 1 - Verifying Trig Identities	
	4.8 Assignment 2 - Trig Identities Worksheet	
	4.9 Assignment - Trigonometric Equations	
	Unit 4: End of Unit Feedback	

Unit 5: Characteristics of Functions

AAL	**AFL**	**AOL**
Lesson 5.1 notes/problem sets	5.1 Assignment Sums and differences of functions	5.3 Assignment Composite Functions (Timed Quiz)
Lesson 5.2 notes/problem sets	5.2 Assignment - Products and Quotients of Functions	Unit 5 Culminating assignment
Lesson 5.3 notes/problem sets	5.4 Assignment Equations and Inequalities	Unit Test
Lesson 5.4 notes/problem sets	5.5 Assignment ARC and IRC Polynomials and Rational Functions	

Lesson 5.5 notes/problem sets	5.6 Assignment ARC and IRC Exponential, Logarithmic and Trig Functions	
Lesson 5.6 notes/problem sets	Unit 5: End of Unit Feedback	

Finals

AOL
Culminating Activity
Final Exam

CONSIDERATION FOR PROGRAM PLANNING

Students learn best when they are engaged in a variety of ways of learning. Guidance and career education courses lend themselves to a wide range of approaches in that they require students to research, think critically, work cooperatively, discuss relevant issues, and learn through practice in a variety of settings. Helping students become self-directed, lifelong learners is a fundamental aim of the guidance and career education curriculum. When students are engaged in active and experiential learning strategies, they tend to retain knowledge for longer periods and develop meaningful skills. Active and experiential learning strategies also enable students to apply their knowledge and skills to real-life issues and situations.

ANTI-DISCRIMINATION EDUCATION IN GUIDANCE AND CAREER EDUCATION

Classroom teachers are the key educators of students who have special education needs. They have a responsibility to help all students learn, and they work collaboratively with special education teachers, where appropriate, to achieve this goal. Special Education Transformation: The Report of the Co-Chairs with the Recommendations of the Working Table on Special Education, 2006 endorses a set of beliefs that should guide program planning for students with special education needs in all disciplines. Those beliefs are as follows: All students can succeed. Universal design and differentiated instruction are effective and interconnected means of meeting the learning or productivity needs of any group of students. Successful instructional practices are founded on evidence-based research, tempered by experience.

PROGRAM CONSIDERATIONS FOR ENGLISH LANGUAGE LEARNERS

Ontario schools have some of the most multilingual student populations in the world. The first language of approximately 20 percent of the students in Ontario's English language schools is a language other than English. Ontario's linguistic heritage includes several Aboriginal languages; many African, Asian, and European languages; and some varieties of English, such as Jamaican Creole. Many English language learners were born in Canada and raised in families and communities in which languages other than English were spoken, or in which the variety of English spoken differed significantly from the English of Ontario classrooms. Other English language learners arrive in Ontario as newcomers from other countries; they may have experience of highly sophisticated educational systems, or they may have come from regions where access to formal schooling was limited. When they start school in Ontario, many of these students are entering a new linguistic and cultural environment.

THE ROLE OF TECHNOLOGY IN THE ENGLISH PROGRAM

Information and communications technologies (ICT) provide a range of tools that can significantly extend and enrich teachers' instructional strategies and support students' language learning. ICT tools include multimedia resources, databases, Internet websites, digital cameras, and word-processing programs. Tools such as these can help students to collect, organize, and sort the data they gather and to write, edit, and present reports on their findings. Information and communications technologies can also be used to connect students to other schools, at home and abroad, and to bring the global community into the local classroom. Whenever appropriate, therefore, students should be encouraged to use ICT to support and communicate their learning.

ACCOMMODATIONS

Accommodations will be based on meeting with parent, teachers, administration and external educational assessment report. The following three types of accommodations may be provided:

- ***Instructional accommodations:*** such as changes in teaching strategies, including styles of presentation, methods of organization, or use of technology and multimedia.
- ***Environmental accommodations:*** such as preferential seating or special lighting.
- ***Assessment accommodations:*** such as allowing additional time to complete tests or assignments or permitting oral responses to test questions.

Other examples of modifications and aids, which may be used in this course, are:

- Provide step-by-step instructions.
- Help students create organizers for planning writing tasks.
- Record key words on the board or overhead when students are expected to make their own notes.
- Allow students to report verbally to a scribe (teacher/ student) who can help in note taking.
- Permit students a range of options for reading and writing tasks.
- Where an activity requires reading, provide it in advance.
- Provide opportunities for enrichment.